Newton's Academy

Mathematics Part - II

Time: 2 Hours **Max. Marks: 40**

Note:

i. *All* questions are compulsory.
ii. Use of calculator is not allowed.
iii. The numbers to the right of the questions indicate full marks.
iv. In case of MCQs [Q. No. 1(A)] only the first attempt will be evaluated and will be given credit.
v. For every MCQ, the correct alternative (A), (B), (C) or (D) with sub-question number is to be written as an answer.
vi. Draw proper figures for answers wherever necessary.
vii. The marks of construction should be clear. Do not erase them.
viii. Diagram is essential for writing the proof of the theorem.

Q.1. (A) For each of the following sub-questions four alternative answers are given. Choose the correct alternative and write its alphabet: **[4]**

i. If $\Delta ABC \sim \Delta DEF$ and $\angle A = 48°$, then $\angle D$ = _______.
(A) 48° (B) 83° (C) 49° (D) 132°

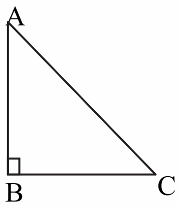
ii. AP is a tangent at A drawn to the circle with center O from an external point P. OP = 12 cm and $\angle OPA = 30°$, then the radius of a circle is _______.
(A) 12 cm (B) $6\sqrt{3}$ cm (C) 6 cm (D) $12\sqrt{3}$ cm

iii. Seg AB is parallel to X-axis and co-ordinates of the point A are (1, 3), then the co-ordinates of the point B can be _______.
(A) (–3, 1) (B) (5, 1) (C) (3, 0) (D) (–5, 3)

iv. The value of $2\tan 45° - 2\sin 30°$ is_______.
(A) 2 (B) 1 (C) $\frac{1}{2}$ (D) $\frac{3}{4}$

(B) Solve the following sub-questions: **[4]**

i. In Δ ABC, $\angle ABC = 90°$, $\angle BAC = \angle BCA = 45°$.
If $AC = 9\sqrt{2}$, then find the value of AB.



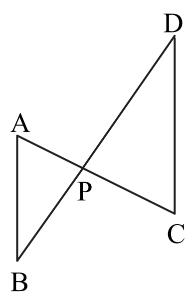
ii. Chord AB and chord CD of a circle with centre O are congruent. If m(arc AB) =120°, then find the m(arc CD).

iii. Find the Y-co-ordinate of the centroid of a triangle whose vertices are (4, –3), (7, 5) and (–2, 1).

iv. If $\sin\theta = \cos\theta$, then what will be the measure of angle θ ?

Q.2. (A) Complete the following activities and rewrite it (any *two*): **[4]**

i. In the above figure, seg AC and seg BD intersect each other in point P. If $\frac{AP}{CP} = \frac{BP}{DP}$, then complete the following activity to prove $\Delta ABP \sim \Delta CDP$.



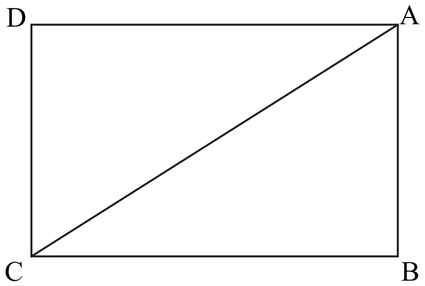
Activity: In ΔAPB and ΔCDP

$\frac{AP}{CP} = \frac{BP}{DP}$ .... ☐

$\therefore$ $\angle APB \cong$ ☐ .... vertically opposite angles

$\therefore$ ☐ $\sim \Delta CDP$....... ☐ test of similarity.

ii. In the above figure, □ABCD is a rectangle. If AB = 5, AC = 13, then complete the following activity to find BC.



Activity:

ΔABC is $\square$ triangle.

$\therefore$ By Pythagoras theorem

$AB^2 + BC^2 = AC^2$

$\therefore$ $25 + BC^2 = \square$ $\therefore$ $BC^2 = \square$

$\therefore$ $BC = \square$

iii. Complete the following activity to prove: $\cot\theta + \tan\theta = \text{cosec}\theta \times \sec\theta$

Activity:

L.H.S. = $\cot\theta + \tan\theta$

$$= \frac{\cos\theta}{\sin\theta} + \frac{\square}{\cos\theta} = \frac{\square + \sin^2\theta}{\sin\theta \times \cos\theta}$$

$$= \frac{1}{\sin\theta \times \cos\theta} \text{........} \because \square = \frac{1}{\sin\theta} \times \frac{1}{\cos\theta}$$

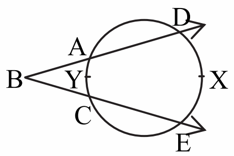
$$= \square \times \sec\theta$$

$\therefore$ L.H.S. = R.H.S.

(B) Solve the following sub-questions (any *four*): **[8]**

i. If $\Delta ABC \sim \Delta PQR$, AB : PQ = 4 : 5 and $A(\Delta PQR) = 125\ cm^2$, then find $A(\Delta ABC)$.

ii.



In the above figure, m(arc DXE) = 105°, m(arc AYC) = 47°, then find the measure of ∠DBE.

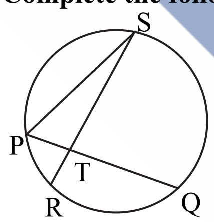
iii. Draw a circle of radius 3.2 cm and centre 'O'. Take any point P on it. Draw tangent to the circle through point P using the centre of the circle.

iv. If $\sin\theta = \frac{11}{61}$, then find the value of $\cos\theta$ using trigonometric identity.

v. In ΔABC, AB = 9 cm, BC = 40 cm, AC = 41 cm. State whether ΔABC is a right-angled triangle or not? Write reason.

Q.3. (A) Complete the following activities and rewrite it (any *one*): **[3]**

i.



In the above figure, chord PQ and chord RS intersect each other at point T. If ∠STQ = 58° and ∠PSR = 24°, then complete the following activity to verify: $\angle STQ = \frac{1}{2}$[m(arc PR) + m(arc SQ)]

Activity:

In ΔPTS,

∠SPQ = ∠STQ – $\square$ $\because$ Exterior angle theorem

$\therefore$ ∠SPQ = 34°

$\therefore$ m(arc QS) = 2 × $\square$° = 68° ....$\because$ $\square$

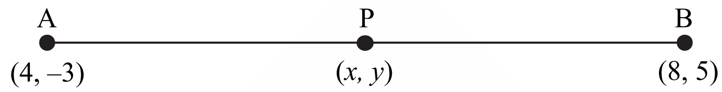
Similarly m(arc PR) = 2∠PSR = $\square$°

$\therefore$ $\frac{1}{2}$[m(arc QS) + m(arc PR)] = $\frac{1}{2} \times \square$° = 58° .... (I) but $\angle STQ = 58°$ .... (II) given

$\therefore$ $\frac{1}{2}$[m(arc PR) + m(arc QS)] = $\boxed{\angle....}$ .... from (I) and (II)

ii. Complete the following activity to find the co-ordinates of point P which divides seg AB in the ratio 3 : 1 where A(4, –3) and B(8, 5).

Activity:



$\therefore$ By section formula,

$x = \frac{mx_2 + nx_1}{\boxed{}}$, $y = \frac{\boxed{}}{m + n}$

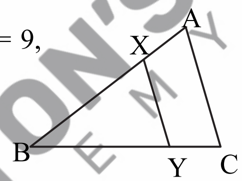
$\therefore$ $x = \frac{3 \times 8 + 1 \times 4}{3 + 1}$, $y = \frac{3 \times 5 + 1 \times (-3)}{3 + 1}$

$\therefore$ $= \frac{\boxed{} + 4}{4}$ $= \frac{\boxed{} - 3}{4}$

$\therefore$ $x = \boxed{}$ $\therefore y = \boxed{}$

(B) Solve the following sub-questions (any *two*): **[6]**

i. In ΔABC, seg XY || side AC. If 2AX = 3BX and XY = 9, then find the value of AC.



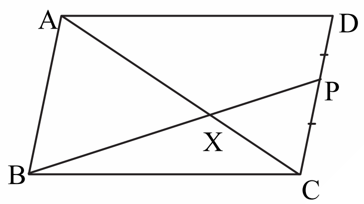
ii. Prove that, "Opposite angles of cyclic quadrilateral are supplementary".

iii. ΔABC ~ ΔPQR. In ΔABC, AB = 5.4 cm, BC = 4.2 cm, AC = 6.0 cm, AB : PQ = 3 : 2, then construct ΔABC and ΔPQR

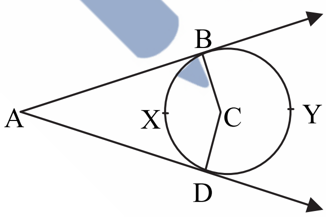
iv. Show that: $\frac{\tan A}{(1 + \tan^2 A)^2} + \frac{\cot A}{(1 + \cot^2 A)^2} = \sin A \times \cos A$.

Q.4. Solve the following sub-questions (any *two*): **[8]**

i. □ABCD is a parallelogram. Point P is the midpoint of side CD. Seg BP intersects diagonal AC at point X, then prove that:
3AX = 2AC



ii.



In the above figure, seg AB and seg AD are tangent segments drawn to a circle with centre C from exterior point A, then prove that: $\angle A = \frac{1}{2}$[m(arc BYD) – m(arc BXD)]

iii. Find the co-ordinates of centroid of a triangle if points D(–7, 6), E(8, 5) and F(2,–2) are the mid-points of the sides of that triangle.

Q.5. Solve the following sub-questions (any *one*): **[3]**

i. If a and b are natural numbers and a > b. If $(a^2 + b^2)$, $(a^2 - b^2)$ and 2ab are the sides of the triangle, then prove that the triangle is right angled.
Find out two Pythagorean triplets by taking suitable values of a and b.

ii. Construct two concentric circles with centre O with radii 3 cm and 5 cm. Construct tangent to a smaller circle from any point A on the larger circle. Measure and write the length of tangent segment. Calculate the length of tangent segment using Pythagoras theorem.